NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION:	BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS		
QUALIFICATION CODE:	07BAMS	**LEVEL:**	7
COURSE CODE:	TSA701S	**COURSE NAME:**	TIME SERIES ANALYSIS
SESSION:	JULY 2023	**PAPER:**	THEORY
DURATION:	3 HOURS	**MARKS**	100

SUPPLEMENTARY/ 2ND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr. Jacob Ong'ala
MODERATOR	Prof. Lilian Pazvakawambwa

INSTRUCTION
1. Answer all the questions 2. Show clearly all the steps in the calculations 3. All written work must be done in blue and black ink

PERMISSIBLE MATERIALS

Non-programmable calculator without cover

THIS QUESTION PAPER CONSISTS OF 3 PAGERS (including the front page)

QUESTION ONE - 20 MARKS

(a) Verify that (max $\rho_1 = 0.5$ nd min $\rho_1 = 0.5$ for $-\infty < \theta < \infty$) for an MA(1) process: $X_t = \varepsilon_t - \theta\varepsilon_{t-1}$ such that ε_t are independent noise processes. **[8 mks]**

(b) State the order of the following ARIMA(p,d,q) processes **[12 mks]**

(i) $Y_t = (1+\phi)Y_{t-1} - \phi Y_{t-2} + e_t$

(ii) $Y_t = Y_{t-1} + e_t - \theta e_{t-1}$

(iii) $Y_t = 5 + e_t - \frac{1}{2}e_{t-1} - \frac{1}{4}e_{t-2}$

(iv) $Y_t = 0.8Y_{t-1} + e_t + 0.7e_{t-1} + 0.6e_{t-2}$

QUESTION TWO - 22 MARKS

Consider AR(3) :$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-2} + \varepsilon_t$ where ε_t is identically independently distributed (iid) as white noise.The Estimates the parameters can be found using Yule Walker equations as

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 1 & \rho_1 & \rho_2 \\ \rho_1 & 1 & \rho_1 \\ \rho_2 & \rho_1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix} \text{ and}$$

$$\sigma_\varepsilon^2 = \gamma_o[(1 - \phi_1^2 - \phi_2^2 - \phi_3^2) - 2\phi_2(\phi_1 + \phi_3)\rho_1 - 2\phi_1\phi_3\rho_2]$$

where

$$\hat{\rho_h} = r_h = \frac{\sum_{t=1}^{n}(X_t - \mu)(X_{t-h} - \mu)}{\sum_{t=1}^{n}(X_t - \mu)^2}$$

$$\hat{\gamma_o} = Var = \frac{1}{n}\sum_{t=1}^{n}(X_t - \mu)^2$$

$$\mu = \sum_{t=1}^{n} X_t$$

Use the data below to evaluate the values of the estimates (ϕ_1, ϕ_2, ϕ_3 and σ_ε^2) **[22 mks]**

t	1	2	3	4	5	6	7	8	9	10
X_t	13	17	15	14	19	22	20	26	32	32

QUESTION THREE - 20 MARKS

A first order moving average $MA(2)$ is defined by $X_t = z_t + \theta_1 z_{t-1} + \theta_2 z_{t-2}$ Where $z_t \sim WN(0, \sigma^2)$ and the $z_t : t = 1, 2, 3..., T$ are uncorrelated.

(a) Find

(i) Mean of the $MA(2)$ **[2 mks]**

(ii) Variance of the $MA(2)$ **[6 mks]**

(iii) Autocovariance of the $MA(2)$ **[8 mks]**

(iv) Autocorrelation of the $MA(2)$ **[2 mks]**

(b) is the MA(2) stationary? Explain your answer **[2 mks]**

QUESTION FOUR - 18 MARKS

Consider the ARMA(1,1) process X_t satisfying the equations $X_t - 0.5X_{t-1} = z_t + 0.4z_{t-1}$ Where $z_t \sim WN(0, \sigma^2)$ and the $z_t : t = 1, 2, 3..., T$ are uncorrelated.

(a) Determine if X_t is stationary **[4 mks]**

(b) Determine if X_t is casual **[2 mks]**

(c) Determine if X_t is invertible **[2 mks]**

(d) Write the coefficients Ψ_j of the $MA(\infty)$ representation of X_t **[10 mks]**

QUESTION FIVE - 20 MARKS

Use the following data shown in the table below to answer the questions that follow.

t	1	2	3	4	5	6	7	8	9	10
Y_t	7	8	9	11	13	16	14	13	18	17

Given $X_t = m_t + R_t$ such that R_t-is the random component following a white noise with a mean of zero and variance of σ^2 and m_t- is the trend,

(a) Estimate the trend using exponential smoothing method with a smoothing parameter $\alpha = 0.68$. **[5 mks]**

(b) Estimate the trend using a centred moving average of order 4 **[6 mks]**

(c) On the same axes, draw the graphs of the original set of data, detrended data (using the exponential smoothing) and detrended data (using the moving average) **[7 mks]**

(d) Which estimates in (a) or (b) above is a better estimate above ((a) or (b)) and why. **[2 mks]**